

1 **Reviewers 1-4**

2 Many thanks to the four reviewers for their patience with the paper, the typos and inconsistencies they identified and
3 the many suggestions provided. We will fix all the typos you pointed out. For example: line 97: Z_i will be deleted,
4 line 119: renumber and clarify remark, line 136: the last ℓ should be H , line 166-167: $F(\mu_0, t)$ should be replaced by
5 $\mu_0((-\infty, t])$, et cetera.

6 **Reviewer 1**

7 **Most of the discussion focusses on Lipschitz weight functions and scant attention is paid to the more interesting**
8 **case of non-Lipschitz weight functions.** As shown in Proposition 1 the pAUC with non-Lipschitz weight function
9 cannot be estimated in general, unless nonatomic measures are assumed. Non-Lipschitz weight functions are handled
10 by the bracketing technique (see Remark 3 on page 5 and the proposed corollary below).

11 **...it is not clear how much of the heavy lifting was already taken care of by [17].** [17] bounds the difference
12 between the empirical functional and its expectation but not the bias (difference between the expectation and the
13 functional evaluated on the underlying law).

14 **Partial AUC is a very well studied problem [19, 20, 21] and indeed corresponds to a discontinuous weight**
15 **function W . The way around is to indeed assume that the underlying law μ does not involve atomic masses...**
16 None of [19, 20, 21] make this assumption, nor would they need it, because they do not address the problem of
17 estimation.

18 **...which is reasonable in most practical situations.** Assuming non-atomic distributions of score functions is a severe
19 restriction and rules out discrete features. For example think of a score function depending on whether a patient has a
20 cough or not.

21 **The paper offers no algorithmic intuitions, nor any explicit algorithms.** [19, 20, 21] all provide elegant algorithms
22 to optimize the empirical error, but they give no generalization guarantees.

23 **Please consider devoting properly stated ways to use Theorem 2, 3 and "sandwiching" results ... to address**
24 **discontinuous weight functions.** We will add the following corollary, which should help to clarify Remark 3 on page
25 5 (with a similar corollary after Theorem 2):

26 **Corollary (of Theorem 3)** Let $\hat{W}, W_{Lip}, W : [0, 1] \rightarrow [0, \infty)$, $\hat{W} \leq W_{Lip} \leq W$ and $\|W_{Lip}\|_{Lip} < \infty$. Then with
27 probability at least $1 - \delta$ in $(\mathbf{X}, \mathbf{Y}) \sim \mu^n$ we have that A_δ implies

$$\forall h \in \mathcal{H}, f_{W,H}((\bar{h}_{\#}\mu)) \geq f_{\hat{W},\ell}(\bar{h}_{\#}\hat{\mu}) - B(n, W_{Lip}, \mathcal{H}, \hat{\mu}, \delta),$$

28 where $B(n, W_{Lip}, \mathcal{H}, \hat{\mu}, \delta)$ is the bound in Theorem 3.

29 Then W would be the (discontinuous) application window and \hat{W} the (discontinuous) training window.

30 **There is prior work...** Many thanks for the reference. We will compare to Theorem 4 therein from the perspectives of
31 rates ($n^{-1/4}$ vs our $n^{-1/2}$), dependence on dimensionality, constants and underlying assumptions.

32 **Notation.** (X, Y) is standard for random labeled instances. We will highlight the distinction between the output of a
33 fixed score function and features.

34 **Reviewer 2**

35 **The authors mention several papers on asymptotic results.** The asymptotic results for L-estimators are not directly
36 applicable to the pAUC. We could give Berry-Esseen-type bounds relying on the Lipschitz properties of the pAUC we
37 identified, but we felt this would go beyond the scope of the paper.

38 **For example, there is a mention of finite sample bounds for specific distributions in some earlier work.** The
39 bounds in [3] are for fixed score functions and special distributions, not so relevant to ML where one wants to select the
40 score function and the distribution is unknown.

41 **Reviewer 3**

42 **AUC users are not quite interested in studying the theoretical properties.** Should the theoretical evidence we
43 provide be disqualified by the lack of numerical experiments? Practitioners could well be interested in the conditions
44 under which their algorithms are protected from overfitting.

45 **Reviewer 4**

46 **For example, line 23 the authors state the estimation could be impossible in the pAUC framework, but without**
47 **bringing support to their assessment.** There should be a reference to Proposition 1 in the sentence following line 23.

48 **I had to reread several times the remark 3 page 4.** We will fix this. Consider also the corollary proposed above in
49 response to Reviewer 1.

50 **Perhaps adding an appendix section that summarizes the definition of $f_{\{W, \ell\}}$, g , etc. would help?** Adding a
51 table of notation in the appendix is a very good idea.

52 Special thanks for reading the proofs in the supplement and detailing the various typos there. We will reorganize the
53 sequence of lemmas in the proof of Proposition 9 and enlarge the remark at the beginning of the proof to clarify the
54 regularity assumptions.

55 **Section 1.1 cites articles but does not really explain the content of the various works.** We will sketch the contents.